Robust Control of Overactuated Autonomous Underwater Vehicle

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Abstract**—*This paper presents a robust control scheme for an over-actuated autonomous underwater vehicle. Since the vehicle is over actuated, a control allocation method is required in conjunction with baseline control law to efficiently distribute control demand to the control effectors. The baseline control law is based on robust H_∞ control. Performances of two control allocation techniques combined with H_∞ control are compared and evaluated. First one is the Explicit Ganging which is an unconstrained control allocation method and, the second one is constrained control allocation method namely Redistributed Pseudo Inverse (RPI) method. Both control allocation techniques are compared using nonlinear six-degree-of-freedom simulation for the cases where some of the control effectors reach their saturation limits. It is shown by simulation results that the RPI method gives an improved performance as compared to the Explicit Ganging for regions where control effectors limit.*

Keywords—Autonomous Underwater Vehicle; Overactuated Systems; Control Allocation; Redistributive Pseudo-Inverse; Explicit Ganging;

I. INTRODUCTION

Role of autonomous underwater vehicles (AUVs) is becoming increasingly significant in the offshore environments. Today, AUVs are being used in deep sea exploration of Oil, gas and sea-floor minerals, underwater cable survey and deployment, the study of marine life and several classified military applications [1]. That's why in recent years focus of many researchers is shifted towards control of AUVs [2-11]. AUV control usually amounts in controlling pitch, heading, surge, sway and depth of the vehicle. Control of these variables is usually provided by propellers. A vast majority of the AUVs use screw-type propellers because of their better efficiency. Other less common examples are the pump-jets [12], and thermal and electric powered gliders [1].

Recently, several control techniques have been used in AUVs. Prominent works in this regard include linear and nonlinear PID control with acceleration feedback [2], LQR control [3], H_∞ control [4-6], State-feedback linearization [7, 8], Integrator Back-stepping [9] and Sliding Mode Control [10].

Along with main control law design, an important consideration in control design of over-actuated systems is that some algorithm is required to distribute control demands to the actuators affecting multiple control channels. This is known as the problem of control allocation. Since many AUVs are designed to be over-actuated to compensate faults and provide better agility, various control allocation schemes are also investigated in recent literature [13-15]. Control allocation methods can be classified into two major categories, i.e. Un-Constrained and Constrained control allocation. As the name implies unconstrained control allocation methods doesn't cater for saturation limits of control effectors. On the other hand a constrained control allocation method incorporates saturation limits specifically. Even in some methods, rate limits, actuator dynamics can also be incorporated. A popular method of control allocation is known as Explicit Ganging [14], which is an unconstrained control allocation method. There are also many other advanced constrained control allocation techniques available in the literature [13, 14], e.g. Direct Allocation, Daisy Chaining, Redistributive Pseudo Inverse (RPI) etc.

This paper presents a robust control scheme for an over-actuated autonomous underwater vehicle. Since the vehicle is over actuated, a control allocation method is required in conjunction with baseline control law to efficiently distribute control demand to the control effectors. The baseline control law is based on robust H_∞ control. Performances of two control allocation techniques combined with H_∞ control are compared and evaluated. First one is the Explicit Ganging which is an unconstrained control allocation method and, the second one is constrained control allocation method namely Redistributed Pseudo Inverse (RPI) method. Both control allocation techniques are compared using nonlinear six-degree-of-freedom simulation for the cases where some of the control effectors reach their saturation limits.

II. VEHICLE MODEL

The schematic of AUV considered in this work is shown in Fig. 1. The sign convention for different channels is selected as follows, positive pitching is about the positive y-axis, positive depth is towards positive z-axis, and positive sway is towards positive y-axis.

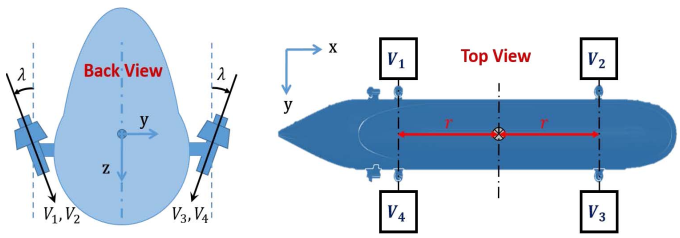

Fig. 1. Schematic of AUV

AUV has eight propellers, four of which are horizontally mounted around the rear-end (horizontal propellers) and remaining four are mounted around the vertical plane, and facing downward with a tilt angle λ (vertical propellers), and separated by a distance *r* from C.G. Horizontal propeller provide control on heading and surge, whereas, vertical propellers provide control on pitch, depth and sway.

The motion of an AUV is exposed to wind, waves, ocean currents along with hydrodynamics and buoyancy forces, and all of these forces affects in six DOF. This restricts the desire for very accurate modeling and control [16], due to unknown dynamics of waves, currents etc. Incorporating major effects of these phenomenons, complete six degrees of freedom dynamics model of the AUV can be written as follows,

$$M_{RB}\dot{q} + M_A\dot{q}_r + C_{RB}q + C_Aq_r + \tau_{RES} = \tau_V + \tau_C + \tau_{DIS} \quad (1)$$

Where

$$M_{RB} = \begin{bmatrix} mI_{3\times3} & -mS(r_G) \\ mS(r_G) & J_0 \end{bmatrix}$$

$$M_{RB} = \begin{bmatrix} O_{3\times3} & -mS(\overline{v}) - mS(\overline{\omega})S(r_G) \\ -mS(\overline{v}) - mS(\overline{\omega})S(r_G) & J_0 \end{bmatrix}$$

$q = [\overline{v}^T, \overline{\omega}^T]^T = [u,v,w,p,q,r]^T$, u,v,w are velocities in body axis, p,q,r are body angular rates, $q_r = [(\overline{v} - \overline{v}_c)^T, \overline{\omega}^T]^T$, $\overline{v}_c$ is sea currents velocity in body axis and assumed constant in earth fixed frame, r_G is position vector from reference point to C.G, m is total mass of AUV, J_0 is inertia tensor of AUV, $S(\cdot)$ is cross product operator, τ_{RES} is combined force and moment vector due to both buoyancy and weight. τ_V is due to viscous hydrodynamic effects, τ_{DIS} is due to disturbances, τ_C is due to propellers, $I_{3\times3}$ and $O_{3\times3}$ are identity and null matrices respectively.

In this work pitch, depth and sway channels are considered, which are controlled by vertical propellers. Since, the system under consideration has four control effectors and three controlled variables (depth, pitch, and sway), thus it is an over-actuated system.

III. CONTROL ARCHITECTURE

Since it is very difficult to accurately model the dynamics of an AUV, thus to mitigate the effects of modeling uncertainties on vehicle performance, a robust control system is preferred. The main advantage of H_∞ controllers over classical linear controls is their maximum robustness, due to this reason it is being used in many applications. Good robustness of H_∞ makes it an ideal candidate for control of an AUV.

The problem of control demand distribution of over-actuated systems can be solved by control allocation methods [17]. Complete architecture of control system incorporating control allocation method is shown in Fig. 2. H_∞ controller generates the desired control forces and moments (τ_d) that must be produced by the propellers to achieve good tracking and stabilization. Than a control allocation algorithm distributes these control demands among propellers and generates propeller commands (u) (in RPM or Volts).

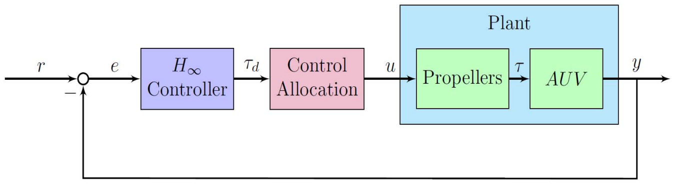

Fig. 2. Control Architecture

The linear constrained control allocation problem is defined as follows: find the control vector u such that,

$$Bu = \tau_d \quad (2)$$

subject to: $u_{\min} \le u \le u_{\max}$

where $B \in \mathbf{R}^{n\times m}$ is control effectiveness matrix, $\tau_d \in \mathbf{R}^n$ is demanded forces vector, $u \in \mathbf{R}^m$ is control command for propellers and u_{min} and u_{max} are lower and upper saturation limits of propellers, respectively. Since we have considered only vertical propellers, so τ_d, B and u are of following forms, respectively.

$$\tau = \begin{bmatrix} F_{depth} & M_{pitch} & F_{sway} \end{bmatrix}$$

$$B = \begin{bmatrix} \cos\lambda & \cos\lambda & \cos\lambda & \cos\lambda \\ r\cos\lambda & -r\cos\lambda & -r\cos\lambda & r\cos\lambda \\ \sin\lambda & \sin\lambda & -\sin\lambda & -\sin\lambda \end{bmatrix}$$

$$u = \begin{bmatrix} V_1 & V_2 & V_3 & V_4 \end{bmatrix}$$

There are two main goals of a control allocation algorithm.

- Determine a unique solution to Eq. 2 when multiple solutions exist.
- Determine the best configuration of control settings when no solution exists.

There are many methods to solve control allocation problems, e.g. Explicit Ganging, Daisy Chaining, Redistributed Pseudo Inverse (RPI), Direct Allocation, and Optimization based Allocation [13, 14]. Each method has its own advantages and disadvantages and thus performs better than other in specific scenarios. Explicit Ganging, is the simplest and most intuitive method, thus usually the first preference for control allocation in over actuated systems. Daisy Chaining is best suited if control effectors are prioritized in some sequential manner. Direct Allocation and

Optimization based Allocation performs well, in most cases but they are computationally very expensive, thus usually cannot be solved online. RPI method has flexibility on computational power requirement by limiting the no. of iterations in each step, it improves performance only if one or more effectors saturates, and it is relatively simpler to implement, thus in this research RPI based control allocation is compared with Explicit Ganging.

IV. H_∞ LOOPSHAPING DESIGN

There are two popular methods, to synthesize H_∞ controller, i.e. Mixed-Sensitivity [18] and Loop Shaping [19], each method has its own merits and demerits. In this research, loop shaping based H_∞ control is designed due to its more intuitive, simple and computationally cheaper controller synthesis procedure. In this paper controller for each channel were designed separately on nominal linearized models, and then validated using complete nonlinear simulation. Fig. 3 shows the control structure for H_∞ loop shaping.

H_∞ loop shaping control synthesis procedure consists of two parts, the first part is to design the weights (W_1 and W_2) such that shaped plant ($G_s = W_2 G W_1$) has desired frequency response. Once loop shaping weights are selected, then find a controller which minimize H_∞ norm of following sensitivity transfer function, to maximize the robustness.

$$\gamma_{\min} = \min_{K_s} \left\| \begin{bmatrix} K_s \\ I \end{bmatrix} (I - G_s K_s)^{-1} M_s^{-1} \right\|_\infty \tag{3}$$

where

G_s is shaped plant tranfer function

M_s is left coprime factor of shaped plant

$(I - G_s K_s)^{-1}$ is sensitivity transfer function

which has exact closed-form solution by [20]

$$\gamma_{\min} = (1 + \rho(XZ))^{1/2} \tag{3}$$

here $\rho(\cdot)$ denotes the spectral radius (maximum eigenvalue)

And X, Z are solutions of following two algebraic Riccati equations.

$$\begin{aligned} &\left(A - BS^{-1}D^T C\right)Z + Z\left(A - BS^{-1}D^T C\right)^T - ZC^T R^{-1} CZ + BS^{-1}B^T = 0 \\ &\left(A - BS^{-1}D^T C\right)^T X + X\left(A - BS^{-1}D^T C\right) - XBS^{-1}B^T X + C^T R^{-1} C = 0 \end{aligned} \tag{4}$$

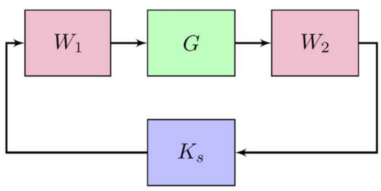

Fig. 4. H∞ Loop Shaping: Shaped Plant and Controller [20]

where,

$$R = I + DD^T, \quad S = I + D^T D$$

For a specified $\gamma > \gamma_{\min}$, the controller can be calculated using following equation, which is slightly modified from [20], the only difference is in a sign of B and D matrices of state space realization of K_s, this is because [20] considered positive feedback in robust stabilization, while we considered negative feedback.

$$K_s = \left[\begin{array}{c|c} A + BF + \gamma^2 \left(L^T\right)^{-1} ZC^T (C + DF) & -\gamma^2 \left(L^T\right)^{-1} ZC^T \\ \hline B^T X & +D^T \end{array}\right] \tag{4}$$

where

$$F = -S^{-1}\left(D^T C + B^T X\right)$$

$$L = \left(1 - \gamma^2\right) I + XZ$$

A. Pitch Control

To design pitch controller following loop shaping weights are selected,

$$W_1 = \frac{705000\ (s + 0.1)}{(s + 0.0001)}, \qquad W_2 = 1$$

H_∞ synthesis yields following 4[th] order controller,

$$K = \frac{4.7489(s + 5.157)(s + 0.3413)(s + 0.162)}{(s + 0.1004)(s + 5.362)(s^2 + 3.671s + 4.799)}$$

Closed loop frequency responses, Sensitivity (S) and Co-Sensitivity (T), are shown in Fig. 4,

B. Depth Control

For depth control following post and pre-filters, are selected for loop shaping,

$$W_1 = \frac{7842.77\ (s + 0.025)}{(s + 0.0001)}, \qquad W_2 = \frac{2.414\ s\ +\ 0.23}{s\ +\ 0.5553}$$

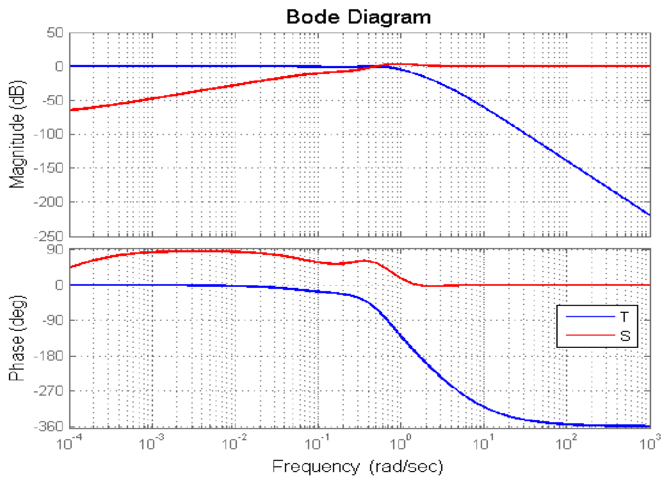

Fig. 3. Closed Loop Bode – Pitch Control

H_∞ synthesis yields following 4[th] order controller

$$K = \frac{3.066\ (s+0.5632)\ (s+0.04466)\ (s+0.03465)}{(s+0.09245)(s+0.02954)(s^2+2.491s+1.983)}$$

Closed loop frequency responses, Sensitivity (S) and Co-Sensitivity (T), are shown in Fig. 5.

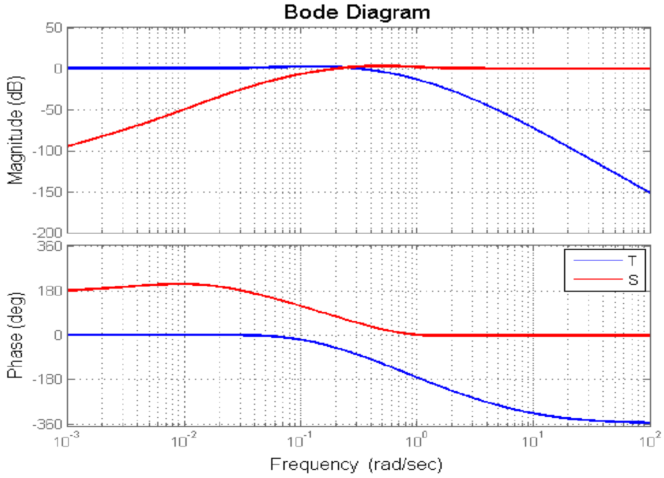

Fig. 5. Closed Loop Bode – Depth Control

C. *Sway Control*

In sway channel following loop shaping weights are selected,

$$W_1 = \frac{438.75\ (s+0.025)}{(s+0.001)}, \qquad W_2 = \frac{2.7475\ (s+0.0182)}{(s+1)\ (s+0.1374)}$$

H_∞ synthesis yields following 3rd order controller,

$$K_r = \frac{0.71344\ (s+0.1969)\ (s+0.01354)}{(s+0.03962)(s^2+0.5552s+0.1176)}$$

Closed loop frequency responses, Sensitivity (S) and Co-Sensitivity (T), are shown in Fig. 6,

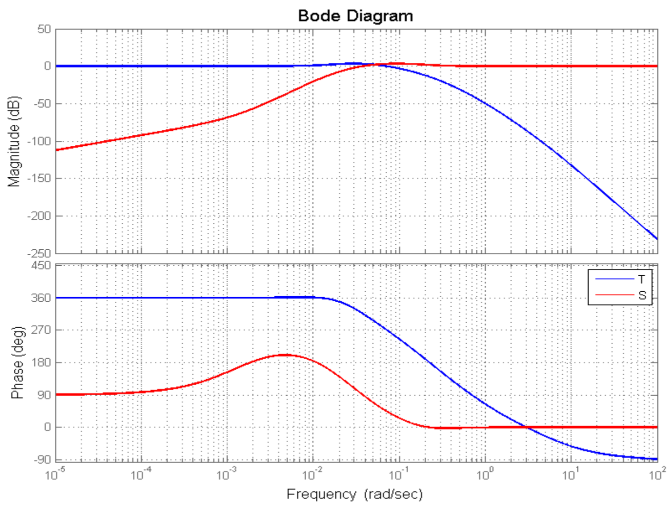

Fig. 6. Closed Loop Bode – Sway Control

V. Control Allocation

In this research Explicit Ganging and RPI methods are compared, for an over-actuated AUV with same baseline robust controllers.

A. *Explicit Ganging*

Due to tilt-angle, each of the vertical propellers can affect multiple controlled variables. A common and most intuitive control mixing strategy is to appropriately add-up pitch, depth and sway control demands to produce control commands for each titled vertical propeller. This scheme is formally known as Explicit Ganging. Some of the major benefits of Explicit Ganging are its intuitiveness, and easier design and implementation, thus it is a common choice of slightly over-actuated system. But in cases where the system is highly over actuated, i.e. very large no. of actuators as compared to degrees of freedom (DOF), it becomes difficult to design and implement this scheme. Also to ensure this to work properly, appropriate budgeting of control demands is needed to efficiently utilize each propeller [13, 14].

In first part of this research, Explicit Ganging based control allocation is used, as shown below,

$$V_1 = \frac{F_{depth}}{4\cos\lambda} + \frac{M_{pitch}}{4r\cos\lambda} + \frac{F_{sway}}{4\sin\lambda}$$
$$V_2 = \frac{F_{depth}}{4\cos\lambda} - \frac{M_{pitch}}{4r\cos\lambda} + \frac{F_{sway}}{4\sin\lambda}$$
$$V_3 = \frac{F_{depth}}{4\cos\lambda} - \frac{M_{pitch}}{4r\cos\lambda} - \frac{F_{sway}}{4\sin\lambda}$$
$$V_4 = \frac{F_{depth}}{4\cos\lambda} + \frac{M_{pitch}}{4r\cos\lambda} - \frac{F_{sway}}{4\sin\lambda}$$

This can also be re-written as,

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} \left(\frac{1}{4\cos\lambda}\right) F_{depth} \\ \left(\frac{1}{4r\cos\lambda}\right) M_{pitch} \\ \left(\frac{1}{4\sin\lambda}\right) F_{sway} \end{bmatrix} \qquad (5)$$

Nevertheless, such scheme has been proven inefficient as situations occur where some propellers become saturated while others remain idle [14]. If such condition is persisted for long duration, overheating or even actuator failure may occur. Additionally, inefficient utilization of propellers also results in poor tracking performance.

B. *Redistributed Pseudo-Inverse Method*

RPI method uses weighted Moore-Penrose pseudo-inverse to find control vector (u), and in case of saturation, commanded forces are redistributed among other propellers using the same pseudo-inverse, and this process continues, until either commanded forces are fully distributed or max number of iterations limit is reached. RPI solution of control allocation problem is given as,

$$u = -c + W^{-1}B^{T}\left[BW^{-1}B^{T} + \varepsilon I_3\right]^{-1}\left[\tau + B_o c\right] \tag{6}$$

Where B_o is original effectiveness matrix, B is the modified effectiveness matrix after each iteration if saturation occurs, τ is demanded forces vector, u is control command for propellers, W is weighting matrix, I_3 is 3×3 identity matrix and c is saturation/offset vector. εI_3 term is added to avoid singularity in pseudo-inverse [16]. Once the solution is computed using Eq. 6, it is checked, if one or more propellers reach saturation, corresponding elements of vector c are set equal to negative of that saturation limit, and corresponding columns of the effectiveness matrix are set equal to zero. This complete process is repeated twice, or more, if required. This algorithm is described in Fig. 7.

VI. Simulation Results

To compare the performance of both schemes, allocation error is defined as below.

$$\text{Allocation Error} = \left\|\tau_d - \tau\right\|_2$$

Where, τ and τ_d are actual and commanded forces and moments vectors, respectively, as shown in Fig. 2. To evaluate the efficiency of RPI method as compared to Explicit Ganging, most control demanding cases were generated, by using the combinations of faster reference commands, sea currents etc. some of these cases are presented here. In CASE–I fast controllers are used in conjunction with rapid maneuvers to hit saturation limits in some propellers. While CASE–II demonstrates the effects of strong sea currents.

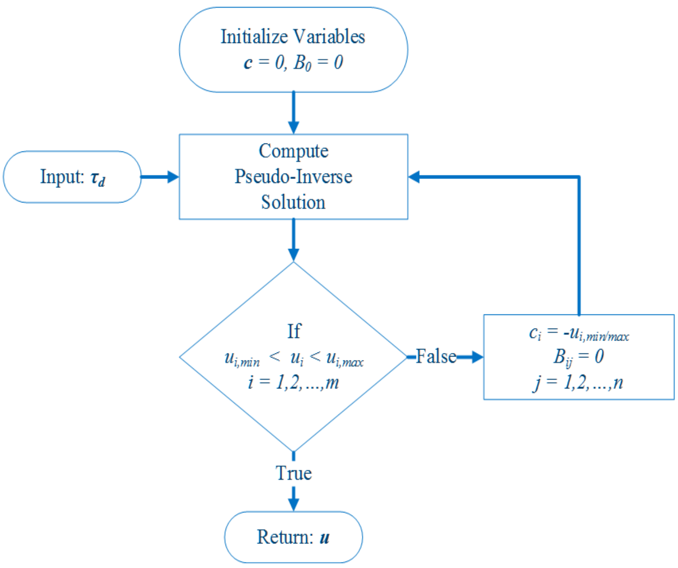

Fig. 7. RPI Method – Flow Chart

A. CASE – 1

In this case, controllers with high cross-over frequency and faster reference commands were used, to intentionally hit saturation in some thrusters, to demonstrate the performance of both algorithms in performance demanding scenarios. As it can be seen clearly from Fig. 8 allocation error is significantly reduced with RPI as compared to Explicit Ganging. Fig. 9 shows significant improvement in pitch tracking, and Fig. 10 and Fig. 11 shows slight improvements in depth and sway tracking, respectively. Fig. 12 shows variations of vertical propellers, and it can be seen clearly that RPI method changed the propeller commands, mainly when some of them are saturated.

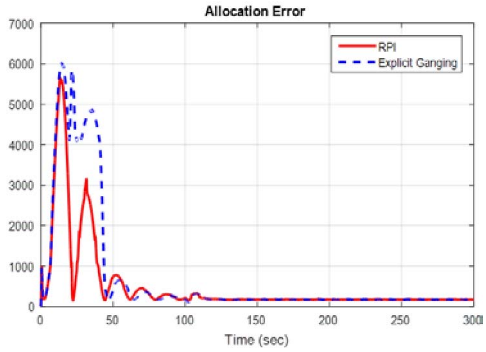

Fig. 8. CASE – 1: Allocation Error

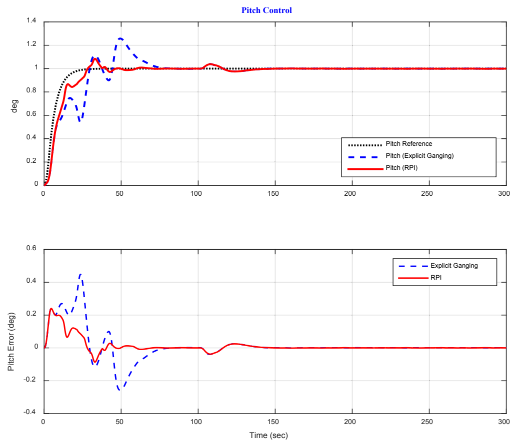

Fig. 9. CASE – 1: Pitch Response

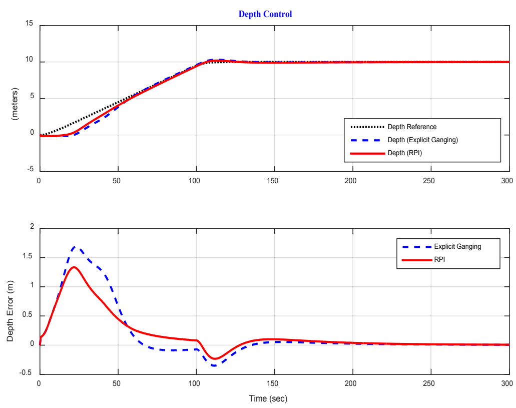

Fig. 10. CASE – 1: Depth Response

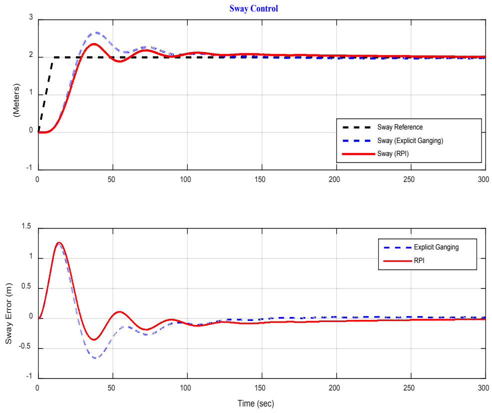

Fig. 11. CASE – 1: Sway Response

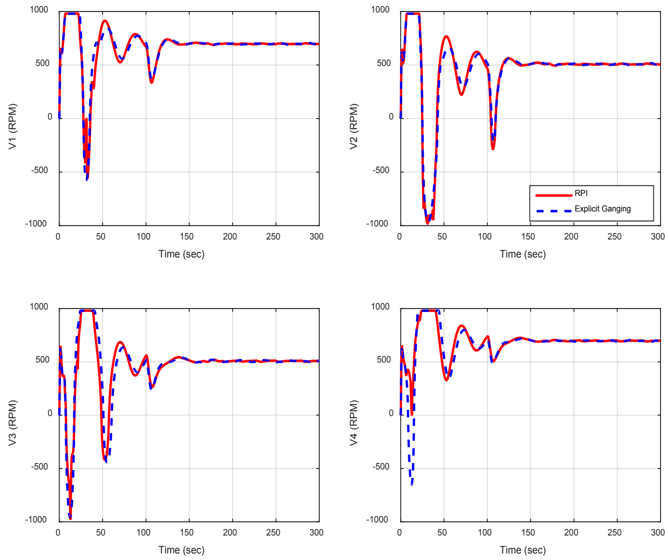

Fig. 12. CASE – 1: Vertical Propellers

B. *CASE – 2*

In this case strong sea currents are applied, to demonstrate the performance of both schemes in strong disturbances. As it can be seen clearly from Fig. 13 allocation error is significantly reduced with RPI method. Fig. 14 shows significant improvement in pitch tracking and Fig. 15 and Fig. 16 shows little improvements in depth and sway tracking, respectively. Fig. 17 shows the variation of vertical propellers.

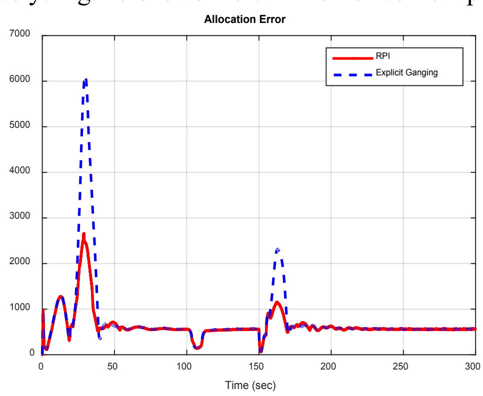

Fig. 13. CASE – 2: Allocation Error

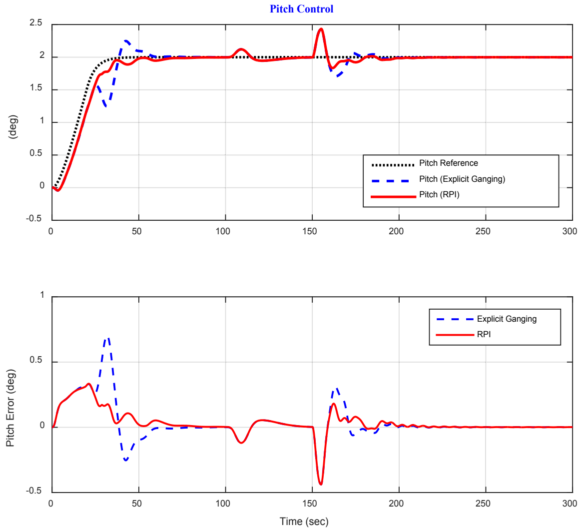

Fig. 14. CASE – 2: Pitch Response

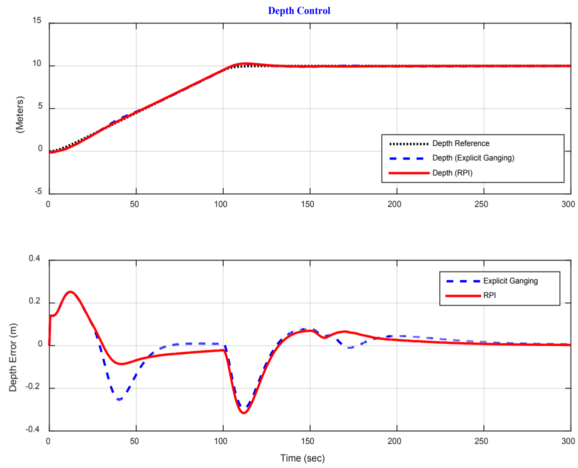

Fig. 15. CASE – 2: Depth Response

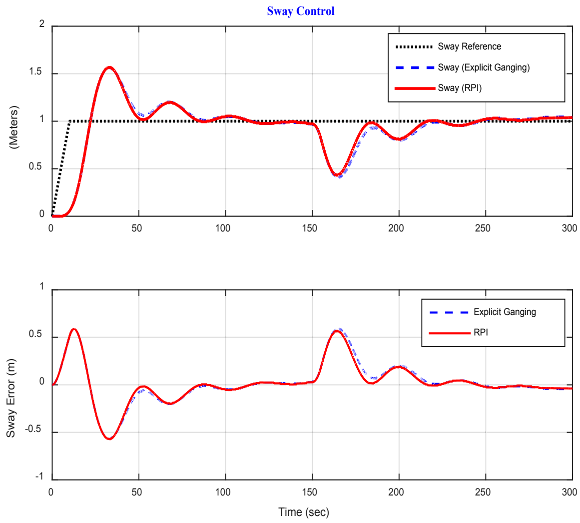

Fig. 16. CASE – 2: Sway Response

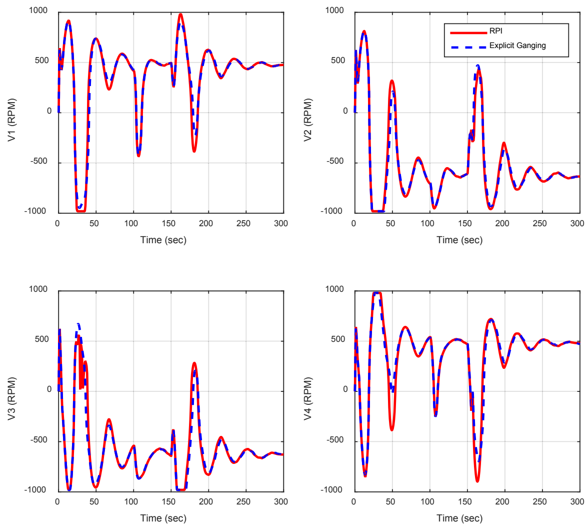

Fig. 17. CASE – 2: Vertical Propellers

VII. Conclusion

In this work, robust H_∞ baseline control laws were synthesized for pitch, depth and sway channels of an over-actuated AUV. Performance of two control allocation techniques, namely, Redistributed Pseudo Inverse (RPI) and Explicit Ganging were compared using nonlinear six-degree-of-freedom simulation, whereas baseline controllers were kept same. Specifically, more demanding cases where some of the control effectors reach their saturation limits were investigated, which usually occurs in rapid maneuvers and strong disturbances. Simulation results demonstrated that the tracking performance of pitch and depth channels were significantly improved when RPI based constrained control allocation was used. Improvement in sway channel, however, was not significant; this is due to lesser control authority of propellers in this channel. Thus it can be concluded that combination of constrained control allocation with robust controller performs better in control demanding conditions.

References

[1] G. Griffiths, *Technology and Applications of Autonomous Underwater Vehicles*, 2003.

[2] K. P. Lindegaard, "Acceleration feedback in Dynamic Positioning System," Ph.D, Department of Engineering and Cybernetics, Norwegian University of Science and Technology, Trondheim, Norway, 2003.

[3] T. Perez, *Ship Motion Control: Course Keeping and Roll Reduction using Rudder and Fins*. Sprfinger-Verlag, London, UK, 2005.

[4] L. Moreira and C. G. Soares, "H2 and H∞ Designs for Diving and Course Control of an Autonomous Underwater Vehicle in Presence of Waves," *IEEE Journal of Oceanic Engineering,* vol. 33, pp. 69-88, 2008.

[5] Z. Feng and R. Allen, "Reduced order H∞ control of an autonomous underwater vehicle," *Control Engineering Practice,* vol. 12, pp. 1511-1520, 2004.

[6] J. Petrich and D. J. Stilwell, "Robust control for an autonomous underwater vehicle that suppresses pitch and yaw coupling," *Ocean Engineering,* vol. 38, pp. 197-204, 2011.

[7] L. Lapierre and D. Soetanto, "Nonlinear path-following control of an AUV," *Ocean Engineering,* vol. 34, pp. 1734-1744, 2007.

[8] U. Ansari and A. H. Bajodah, "Robust generalized dynamic inversion based control of autonomous underwater vehicles," *Proceedings of the Institution of Mechanical Engineers, Part M: Journal of Engineering for the Maritime Environment,* 2017.

[9] T. I. Fossen and J. P. Strand, "A Tutorial on Nonlinear Backstepping: Application to Ship Control," *Modeling, Identification and Control,* vol. 20(2), pp. 83-135, 1999.

[10] A. J. Healey and D. Lienard, "Multivariable sliding mode control for autonomous diving and steering of unmanned underwater vehicles," *IEEE Journal of Oceanic Engineering,* vol. 18, pp. 327-339, 1993.

[11] G. Antonelli, S. Chiaverini, N. Sarkar, and M. West, "Adaptive control of an autonomous underwater vehicle: experimental results on ODIN," *IEEE Transactions on Control Systems Technology,* vol. 9, pp. 756-765, 2001.

[12] A. M. Tongue, "The UK military UUV program- a program update. Proceedings of UUVS 2000," New Malden, UK, 2000, pp. 143-148.

[13] T. A. Johansen and T. I. Fossen, "Control allocation - A survey," *Automatica,* vol. 49, pp. 1087-1103, 2013.

[14] M. Oppenheimer, D. Doman, and M. Bolender, "Control Allocation," in *Control System Applications*, 2nd ed: CRC Press, 2010.

[15] J. C. Virnig and D. S. Bodden, "Multivariable control allocation and control law conditioning when control effectors limit," presented at the Guidance, Navigation, and Control Conference, 1994.

[16] T. I. Fossen, *Handbook of marine craft hydrodynamics and motion control*: John Wiley & Sons, 2011.

[17] M. Bodson, "Evaluation of Optimization Methods for Control Allocation," *Journal of Guidance, Control, and Dynamics,* vol. 25, pp. 703-711, 2002.

[18] J. C. Doyle, K. Glover, K. P. P., and B. A. Francis, "State-space solutions to standard H2 and H∞ control problems," *IEEE Transactions on Automatic Control,* vol. 34(8), pp. 831–847, 1989.

[19] K. Glover and D. McFarlane, "Robust stabilization of normalized coprime factor plant descriptions with H∞ bounded uncertainty," *IEEE Transactions on Automatic Control,* vol. 34(8), pp. 821–830, 1989.

[20] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control: Analysis And Design*, 2nd ed.: John Wiley & Sons, 2001.